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Illustrative Evaluation Index for Haptic Interfaces using Confusion Matrices

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***Abstract*—For researchers of haptic interfaces, evaluation of the perceptual similarity between virtual and real haptic stimuli has long been a serious problem. One of the most commonly employed evaluation methods is an identification task where assessors identify the type of randomly presented stimuli among multiple candidates. The results of this method are summarized as confusion matrices. We developed a method that allocates all virtual and real stimuli in a perceptual space. The spatial distribution of the stimuli allows us to visually understand the perceptual relationships between the stimuli. A brief validation confirmed that the proposed method is effective in evaluating the perceptual similarity between virtual and real stimuli.**

I. Introduction

Assessment of the stimuli displayed to users of haptic interfaces is vital. Various methods have been employed for this purpose, for example, comparing the similarity of physical quantities, such as force or skin deformation, between virtual and real stimuli (e.g. [1], [2]). However, studying the physical similarity is not sufficient, and researchers often exploit the evaluation of perceptual similarity. In some cases, assessors evaluate the degree of perceptual reality using numerical scales. In other cases, they conduct sensory evaluations with certain criteria such as the perceived roughness of textures or hardness of objects. For each criterion, the similarity between virtual and real stimuli is discussed. Questionnaire-based reality evaluation for virtual reality systems (e.g., [3], [4]) is also applicable to haptic interfaces. Such methods have their advantages and disadvantages; however, a general method does not exist. Researchers tend to select methods appropriate for their own purposes.

Identification tasks: Many haptic researchers have chosen identification tasks owing to their simplicity and objectivity (e.g., [5], [6], [7], [8]). In these tasks, a blindfolded assessor chooses a stimulus that feels most similar to the randomly presented stimulus among n candidates (n-alternative forced choice). Then, the goodness of haptic interfaces is discussed on the basis of correct response ratios.

Two types of experiments are commonly performed in identification tasks. In one, the virtual-to-real identification task, assessors choose the real stimulus that feels closest to the virtual stimulus presented to them. In the other, the real-to-real identification task, assessors respond to a real stimulus. The results of these two experiments are compared to assess haptic interfaces and stimuli. Some researchers focus on only the correct responses; however, wrong answer ratios also provide important information.

Confusion matrices: Confusion matrices are often used to show the results of identification tasks. We acquire two matrices from the two types of experiments mentioned above. One problem is that it is not easy to understand and compare the meanings of wrong answer ratios between these two matrices. Tables I and II show the examples of matrices established by four stimuli. In confusion matrices, diagonal elements indicate correct response ratios and the others are confusion ratios. r_i and v_i ($i = 1, ..., 4$) are types of real and virtual stimuli, respectively. v_i is a virtual stimulus that mimics r_i. P_{sisj} is a probability at which assessors classify s_i as s_j.

First, the real-to-real identification task table (Table I) shows that because P_{r1r1} and P_{r2r2} are high, r_1 and r_2 are somewhat easily identified. On the other hand, P_{r3r3} and P_{r4r4} are relatively small and P_{r3r4} and P_{r4r3} are considerably high. Hence, r_3 and r_4 are likely to be confused. Second, the virtual-to-real identification task table (Table II) shows that the overall correct response ratios are lower than those of the real-to-real identification task. Thus, virtual stimuli incompletely capture the characteristics of real stimuli, and the confusion ratios elevate. Specifically, v_3 and v_4 are problematic because P_{v3r3} and P_{v4r4} are significantly smaller than P_{r3r3} and P_{r4r4}, and P_{v3r4} and P_{v4r3} are larger than P_{r3r4} and P_{r4r3}. Also, since P_{v2r3} and P_{v3r2} are larger than P_{r2r3} and P_{r3r2}, v_2 and v_3 feel more similar than the combination of r_2 and r_3. Finally, P_{r1r1} and P_{v1r1} are the same value, which indicates that v_1 well represents r_1.

It is informative to collectively consider both the correct and wrong answer ratios. However, for those who are not familiar with identification tasks, it is not easy to capture the meanings of the wrong answer ratios of the two matrices. Thus, wrong answer ratios, which actually include thoughtful information, tend to be ignored. Correct and wrong answer ratios collectively tell us what improvements are necessary for better haptic interfaces.

Objective: The objective of this study is to integrate two types of confusion matrices and construct an illustrative index for intuitive understanding, in which all real and virtual stimuli are placed in a perceptual space. Geometric distances in this space correspond to the perceptual dis-

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TABLE I
REAL-TO-REAL IDENTIFICATION TASK: CONFUSION MATRIX

		Answered			
		r_1	r_2	r_3	r_4
Presented	r_1	P_{r1r1}=0.8	P_{r1r2}=0.05	P_{r1r3}=0.08	P_{r1r4}=0.08
	r_2	P_{r2r1}=0.05	P_{r2r2}=0.8	P_{r2r3}=0.08	P_{r2r4}=0.08
	r_3	P_{r3r1}=0.05	P_{r3r2}=0.05	P_{r3r3}=0.7	P_{r3r4}=0.2
	r_4	P_{r4r1}=0.05	P_{r4r2}=0.05	P_{r4r3}=0.2	P_{r4r4}=0.7

TABLE II
VIRTUAL-TO-REAL IDENTIFICATION TASK: CONFUSION MATRIX

		Answered			
		r_1	r_2	r_3	r_4
Presented	v_1	P_{v1r1}=0.8	P_{v1r2}=0.07	P_{v1r3}=0.07	P_{v1r4}=0.07
	v_2	P_{v2r1}=0.05	P_{v2r2}=0.6	P_{v2r3}=0.2	P_{v2r4}=0.15
	v_3	P_{v3r1}=0.05	P_{v3r2}=0.2	P_{v3r3}=0.5	P_{v3r4}=0.25
	v_4	P_{v4r1}=0.05	P_{v4r2}=0.2	P_{v4r3}=0.25	P_{v4r4}=0.5

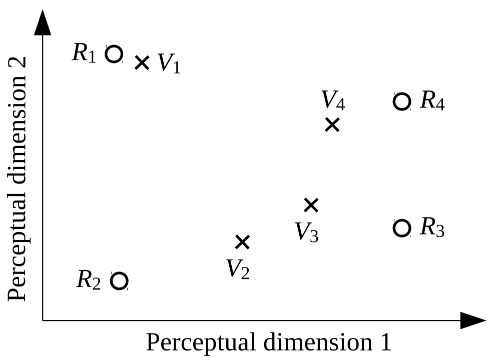

Fig. 1. Real and virtual stimuli placed in perceptual space

similarities of the stimuli. The illustrative index allows us to easily obtain the results of identification tasks. There are two technical challenges to establishing such an index. First, two confusion matrices are based on two separately conducted experiments. These matrices must therefore be combined to maintain consistency. Second, assessors do not directly compare pairs of virtual stimuli, whereas they do compare pairs of real stimuli and pairs of virtual and real stimuli in real-to-real and virtual-to-real identification tasks, respectively. Thus, the perceptual distances between virtual stimuli must be computed from unsatisfactory information.

Novelty of this study: To the best of our knowledge, this work is the first attempt to construct a perceptual space of real and virtual stimuli from confusion matrices whereas some measures were established for such matrices. Powerful methods that compute inter-stimuli psychological distances from confusion matrices are the similarity choice model [9], [10] and overlap model [11], [12]. However, these distances are not computed by two confusion matrices that include different stimulus elements. In our problem, two matrices including different elements must be integrated.

We use a multidimensional scaling approach to construct a spatial distribution of stimuli. This approach has been used to show the perceptual distances or equivalences between haptic stimuli [13], [14], [15], [16]. For the computation of this approach, perceptual distances between stimuli are required. They were typically acquired by clustering tasks [13], [14] or scaling tasks [15], [16]. In the former tasks, participants clustered the stimuli into several groups based on their similarities. In the latter tasks, the dissimilarity between two stimuli are specified using a numerical scale. On the other hand, our proposed method allows us to compute a multidimensional scaling method from the results of identification tasks.

II. ILLUSTRATIVE INDEX FOR IDENTIFICATION TASKS

The illustrative index allocates real and virtual stimuli in a perceptual space, in which geometric distances correspond to perceptual dissimilarities. Fig. 1 shows the index constructed by Tables I and II. R_i and V_i are locations of r_i and v_i in the perceptual space. Since R_1 and R_2 are located far from the other real stimuli, r_1 and r_2 are easily identified. R_3 and R_4 are closely located and, r_3 and r_4 are frequently confused. The virtual stimuli are distributed inside of the real stimuli, indicating that the correct response ratios of the virtual-to-real identification task are smaller than those of the real-to-real identification task. V_2, V_3, and V_4 are perceptually close to each other and their confusion ratios are high. On the other hand, V_1 and R_1 are very close, which suggests that the confusion ratios between v_1 and r_i ($i = 2, 3, 4$) are almost the same as those between r_1 and r_i ($i = 2, 3, 4$). V_2 and R_2 are far from each other and dissimilar; thus, v_2 needs to be improved such that it feels more similar to r_2. As mentioned above, the introduction of the illustrative index enhances the understanding of perceptual relationships between stimuli.

To construct the index, we first compute the perceptual distances between all stimuli. Different methods are used to compute the distances between pairs of real stimuli, real and virtual stimuli, and virtual stimuli. We then apply a multidimensional scaling method on the computed distances. The stimuli are located in a two- or three-dimensional space so that their distances are well maintained.

A. Computation of inter-stimuli distance

Let D_{sisj} be the perceptual distance between s_i and s_j. Given that $D_{sisj} = D_{sjsi}$, to compute the multidimensional scaling method, the upper triangular part of the distance matrix in Fig. 2 is necessary. D_{rirj} and D_{vivj} are computed from the results of real-to-real and virtual-to-real identification tasks, respectively. In order to integrate the two tasks while maintaining consistency, D_{rivj} is computed by combining the results of both tasks.

1) Model of internal response by signal detection theory: In detection theory, identification of a stimulus among n candidates is modeled as an identification of stimuli on perceptual multi-dimensions. The human internal response to a physical stimulus is expressed as a normal distribution, as shown in Fig. 3. The internal response toward a physical stimulus s_i follows a joint probability density function

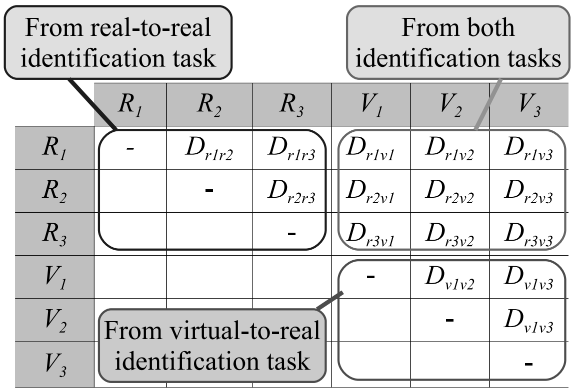

Fig. 2. Distance matrix of real and virtual stimuli (three virtual and real stimuli)

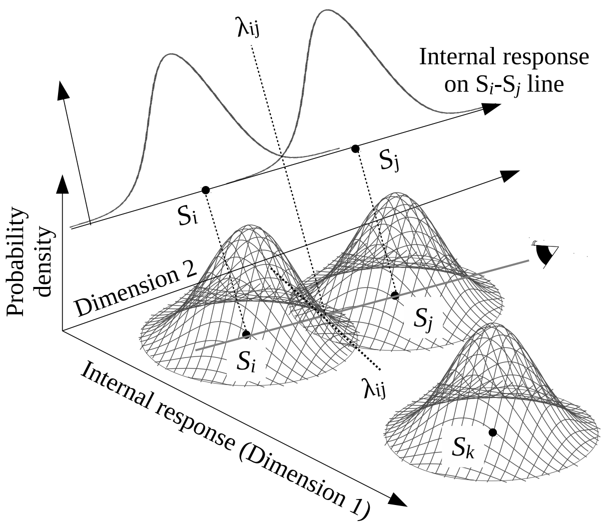

Fig. 3. Internal responses of multiple stimuli and application of constant ratio rule

of multiple variables (two variables in the figure) with the center of distribution S_i. These variables are mutually independent and construct the perceptual space of stimuli. In this model, when the response toward s_i appears closer to S_i than S_j, the stimulus is classified or recognized as s_i. The border of this judgment is called criterion λ_{ij}. We assume the equality of variance of distributions, which is commonly accepted. The unit of each dimensional variable is set to be equal to this variance.

We deal with stimuli on multi-dimensions, following the constant ratio rule [17], [18]. Applying this rule to two arbitrary stimuli s_i and s_j, their probabilistic and perceptual relationships are discussed on the $S_i - S_j$ line. As shown in Fig. 3 (upper left), the internal responses of the two stimuli are simplified. We estimate their perceptual distance as $D_{sisj} = S_j - S_i$ on this line.

2) Distance between real stimuli: Fig. 4 shows the internal responses toward a real stimulus r_i on the $R_i - R_j$

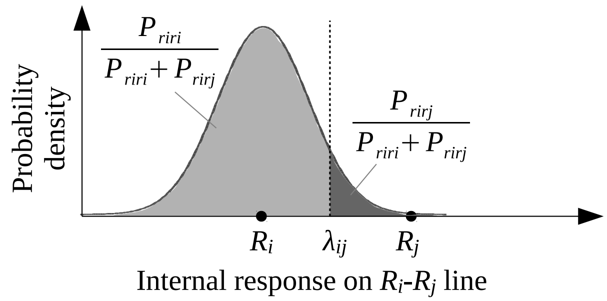

Fig. 4. Internal response toward r_i on R_i-R_j line

line. On this line,

$$\lambda_{ij} - R_i = Z(\frac{P_{riri}}{P_{riri} + P_{rirj}}) \tag{1}$$

holds, where $Z(p)$ is the z-score of probability p. $Z(p)$ is described by

$$p = g(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(\frac{-x^2}{2}) dx \tag{2}$$

$$Z(p) = g^{-1}(p). \tag{3}$$

Similarly, $R_j - \lambda_{ij}$ is given by

$$R_j - \lambda_{ij} \quad = \quad Z(\frac{P_{rjrj}}{P_{rjrj} + P_{rjri}}). \tag{4}$$

From these equations, the perceptual distance between two real stimuli is determined by

$$\begin{aligned} D_{rirj} &= |R_j - R_i| \\ &= |Z(\frac{P_{riri}}{P_{riri} + P_{rirj}}) + Z(\frac{P_{rjrj}}{P_{rjrj} + P_{rjri}})|. \end{aligned} \tag{5}$$

3) Distance between real and virtual stimuli: In a virtual-to-real identification task in which assessors classify virtual stimuli into real stimuli, the internal responses of stimuli are expressed as shown in Fig. 5. When virtual stimulus v_i is presented and its internal response is closer to R_i, the stimulus is classified as r_i. On the other hand, if the response is closer to R_j, then it is classified as r_j. The border of this judgment is described as λ_{ij}. Looking at this internal response on the line that passes V_i and the center of R_i and R_j, the response to v_i is expressed as shown in Fig. 6. $\lambda_{ij} - V_i$ is described by

$$\lambda_{ij} - V_i = |Z(\frac{P_{viri}}{P_{viri} + P_{virj}})|. \tag{6}$$

On the line,

$$\begin{bmatrix} 1 & 1 \\ \text{-}1 & 1 \end{bmatrix} \begin{bmatrix} R_i - V_i \\ R_j - V_i \end{bmatrix} = \begin{bmatrix} 2(\lambda_{ij} - V_i) \\ D_{rirj} \end{bmatrix} \tag{7}$$

holds. The upper column holds with the assumption that λ_{ij} locates at the center of R_i and R_j. The distances between virtual and real stimuli are given by

$$\begin{bmatrix} D_{viri} \\ D_{virj} \end{bmatrix} = \begin{bmatrix} |R_i - V_i| \\ |R_j - V_i| \end{bmatrix}. \tag{8}$$

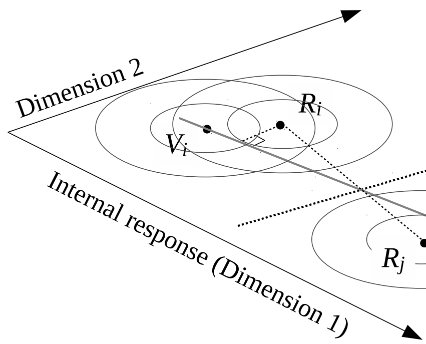

Fig. 5. Internal responses toward v_i, r_i, and r_j. For visual clarity, probability densities are not shown.

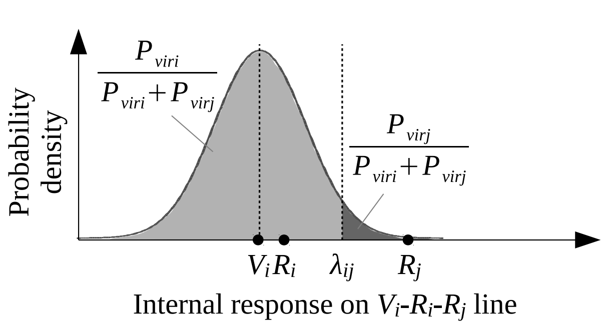

Fig. 6. Internal response toward v_i on the V_i-R_i-R_j line

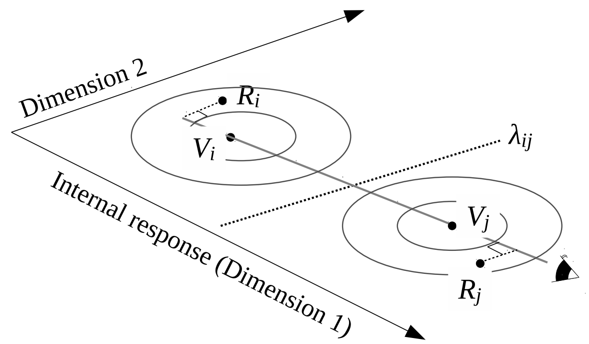

Fig. 7. Internal response toward V_i and V_j

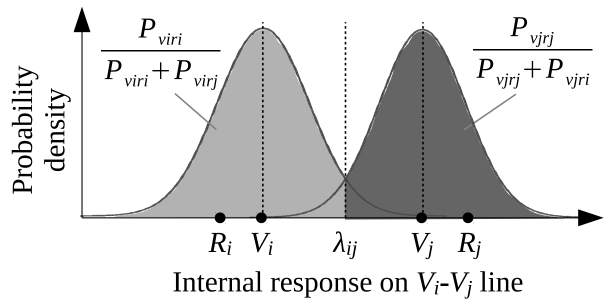

Fig. 8. Internal response toward V_i and V_j on the V_i-V_j line

These distances are given by solving (7). D_{rirj} is determined in the above section from the results of the real-to-real identification task. Thus, D_{viri} and D_{virj} are computed by the results of both identification tasks; this combines the two identification tasks and maintains their consistency. In this computation, R_i and R_j are projected on the $V_i - R_i - R_j$ line. Hence, D_{viri} and D_{virj} tend to be underestimated.

4) Distance between virtual stimuli: Finally, we compute the distances between virtual stimuli. In identification tasks, assessors do not directly compare virtual stimuli. However, from the results of the virtual-to-real identification task, the distances between virtual stimuli can be computed. Fig. 7 shows the internal responses toward v_i and v_j and R_i and R_j. We look at this space on a line that goes though V_i and V_j. Fig. 8 shows the responses toward v_i and v_j and the projections of R_i and R_j on this line. Here,

$$\lambda_{ij} - V_i = Z(\frac{P_{viri}}{P_{viri} + P_{virj}}) \tag{9}$$

holds. Similarly,

$$V_j - \lambda_{ij} = Z(\frac{P_{vjrj}}{P_{vjrj} + P_{vjri}}) \tag{10}$$

holds. The distances between two virtual stimuli are computed by these equations, as follows:

$$\begin{aligned} D_{vivj} &= |V_j - V_i| \\ &= |Z(\frac{P_{viri}}{P_{viri} + P_{virj}}) + Z(\frac{P_{vjrj}}{P_{vjrj} + P_{vjri}})|. \end{aligned} \tag{11}$$

B. Multidimensional scaling method

We use a multidimensional scaling method to compute the distribution of stimuli in the perceptual space from their perceptual distances. This method determines the coordinates of events (stimuli) in a space while maintaining the given distances between them. The number of dimensions can be set arbitrarily; however, in general, larger dimensions represent the distances better. Considering that our index is illustrative, two or three dimensions are preferable. In the present study, we use the metric multidimensional scaling developed by Torgerson [19].

III. EXAMPLE AND BRIEF VALIDATION

We validate the method by checking the consistency between the confusion matrix examples and computed spatial distribution of stimuli. We present examples of the three-stimulus cases. Table III shows the confusion matrix of the real-to-real identification task. We use this table to compute the stimulus distribution for various matrices of the virtual-to-real identification task. In the following examples, if the distance between two stimuli is 2, they are correctly classified at 84% ($2 \cdot Z(0.84) = 2.0$). As is the nature of multidimensional scaling methods, the orthogonal rotation of the stimuli's coordinates is indeterminate.

Example 1 (Shrunk space): First, we compute the stimuli distribution when the confusion matrix of the virtual-to-real task is given as in Table IV, where the correct response ratios are smaller than those for the real-to-real task. The wrong answer ratios are balanced. In this

TABLE III
REAL-TO-REAL IDENTIFICATION TASK: CONFUSION MATRIX (EXAMPLES 1–5)

Present	Answered r_1	r_2	r_3
r_1	0.8	0.1	0.1
r_2	0.1	0.8	0.1
r_3	0.1	0.1	0.8

TABLE IV
EXAMPLE 1 (SHRUNK): CONFUSION MATRIX OF VIRTUAL-TO-REAL IDENTIFICATION

Present	Answered r_1	r_2	r_3
v_1	0.7	0.15	0.15
v_2	0.15	0.7	0.15
v_3	0.15	0.15	0.7

TABLE V
EXAMPLE 2 (DISTORTED): CONFUSION MATRIX OF VIRTUAL-TO-REAL IDENTIFICATION

Present	Answered r_1	r_2	r_3
v_1	0.8	0.1	0.1
v_2	0.1	0.7	0.2
v_3	0.1	0.2	0.7

TABLE VI
EXAMPLE 3 (EXPANDED): CONFUSION MATRIX OF VIRTUAL-TO-REAL IDENTIFICATION

Present	Answered r_1	r_2	r_3
v_1	0.9	0.05	0.05
v_2	0.05	0.9	0.05
v_3	0.05	0.05	0.9

TABLE VII
EXAMPLE 4 (SAME): CONFUSION MATRIX OF VIRTUAL-TO-REAL IDENTIFICATION

Present	Answered r_1	r_2	r_3
v_1	0.8	0.1	0.1
v_2	0.1	0.8	0.1
v_3	0.1	0.1	0.8

TABLE VIII
EXAMPLE 5 (CONFUSED): CONFUSION MATRIX OF VIRTUAL-TO-REAL IDENTIFICATION

Present	Answered r_1	r_2	r_3
v_1	0.8	0.1	0.1
v_2	0.1	0.4	0.5
v_3	0.1	0.5	0.4

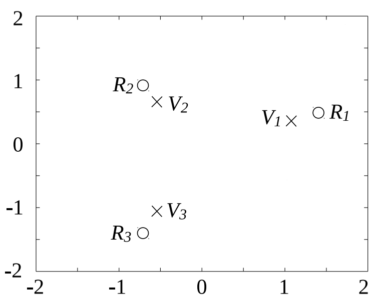

Fig. 9. Example 1 (shrunk): Stimuli distribution

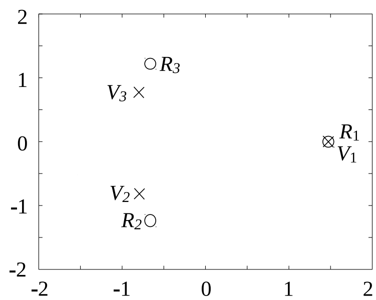

Fig. 10. Example 2 (distorted): Stimuli distribution

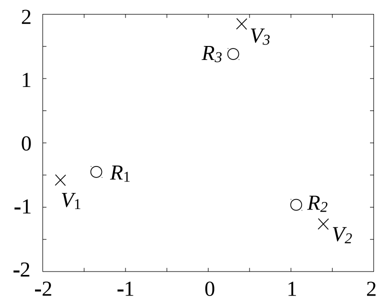

Fig. 11. Example 3 (expanded): Stimuli distribution

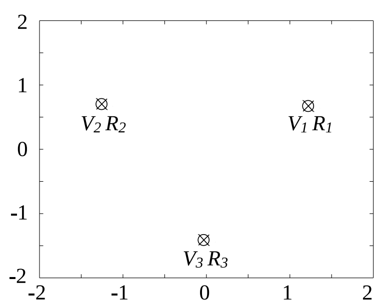

Fig. 12. Example 4 (same): Stimuli distribution

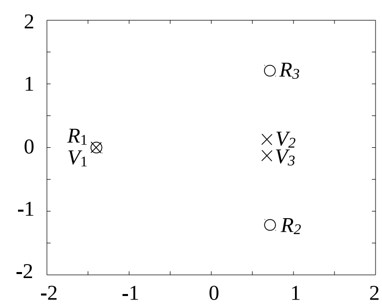

Fig. 13. Example 5 (confused): Stimuli distribution

case, the distribution of virtual and real stimuli should be geometrically similar and only their areas should be different. Naturally, the distribution of virtual stimuli should be smaller than that of real stimuli.

Fig. 9 shows the distribution of stimuli computed using Tables III and IV. As expected, the distributions of real and virtual stimuli are similar, and the area for virtual stimuli is smaller than that for real stimuli.

Example 2 (Distorted space): As shown in Table V, we decrease P_{v2r2} and P_{v3r3}, while keeping P_{v1r1} as high as P_{r1r1}. Accordingly, we increase P_{v2r3} and P_{v3r2}. In this case, V_1 should be located on the same coordinate as R_1 because $P_{r1ri} = P_{v1ri}$ $(i = 2, 3)$, which means that V_1 is the same as R_1 in terms of the relationships with other real stimuli. Also, V_2 and V_3 should approach R_3 and R_2, respectively, because V_2 and V_3 are more likely to be confused than R_2 and R_3.

Fig. 10 shows the distribution of stimuli computed using Tables III and V. R_1 and V_1 are placed together while V_2 and V_3 are placed closer than R_2 and R_3. These tendencies are consistent with the above expectations. As a result, the distribution of virtual stimuli is distorted compared with that of real stimuli.

Example 3 (Expanded space): As shown in Table VI, we increase the correct response ratios of the virtual-to-real identification task, and the wrong answer ratios are balanced. In this case, the distribution of virtual stimuli should be larger than that of real stimuli. The polygons comprising virtual and real stimuli should be similar because the proportions of wrong answer ratios are equal between both identification tasks.

Fig. 11 shows the distribution of stimuli computed using Tables III and VI. The polygon comprising virtual stimuli is located outside that of real stimuli. Both polygons are geometrically similar. As a result, the distribution of virtual stimuli appears to be expanded compared with that of real stimuli. This could occur when the features of real stimuli are well represented and even emphasized by virtual stimuli.

Example 4 (Same matrix): When the confusion matrices of both identification tasks are identical, the allocations of virtual and real stimuli in a perceptual space should also be the same. Fig. 12 shows the perceptual space computed under this condition, using Tables III and VII. The coordinates of V_i and R_i match.

Example 5 (Confused case): In the above examples, the diagonal elements of the confusion matrix of the virtual-to-real identification task are larger than the others. In other words, v_i is most frequently classified as r_i. However, such an ideal case does not always exist in actual experiments. For example, as shown in Table VIII, v_2 and v_3 can be wrongly classified as r_3 and r_2, respectively. In this case, V_2 should be closer to R_3 than to R_2. Also, V_3 should be located near R_2.

Fig. 13 shows the perceptual space computed using Tables III and VIII. Consistent with the above expectations, V_2 is closer to R_3 than to R_2 and V_3 is closer to R_2. This indicates that stimuli 2 and 3 are totally confused.

Summary: The above examples confirm the consistency between the confusion matrices and acquired perceptual space.

IV. Conclusions

In this study, we developed an illustrative index for evaluating haptic interfaces, which helped us understand the perceptual similarities between virtual and real haptic stimuli. The index was constructed from the results of identification tasks, which are commonly used for evaluating haptic interfaces. We computed the perceptual distances of stimuli using detection theory. The first challenge was integration of the two tasks: real-to-real and virtual-to-real identification tasks. We computed the distances by combining the results of the two tasks while maintaining their consistency. Another challenge was the acquisition of distances between virtual stimuli, which assessors do not typically compare in identification tasks. Finally, we used a multidimensional scaling method and allocated all virtual and real stimuli in the perceptual space. This allowed us to visually capture how well the virtual stimuli were represented in terms of perceptual similarities. We validated our method by applying it to some confusion matrix examples and confirmed the consistency between the computed spatial distributions of stimuli and the confusion matrices. Through further statistical validations, we hope to construct an even more reliable and useful index for haptic interfaces.

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