

Effects of Mechanical Parameters on Hardness Experienced by Damped Natural Vibration Stimulation

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Abstract—Humans can perceive the hardness of an object from the damped natural vibration produced by tapping its surface. This damped natural vibration can be used in haptic rendering to effectively present a sense of the hardness of an object. Although its frequency is known to influence the subjective hardness, the effects of the viscosity or decay rate have yet to be studied. We researched the contributions of the mass, viscosity, and stiffness to the hardness perception using a commercial force display and psychophysical experiments, where only the vibratory displacement was presented. As a result, the stiffness was found to have a positive effect on the subjective hardness, similar to the usual haptic rendering of an object. The mass of the object was negatively correlated with the subjective hardness. These results indicated that the vibratory frequency was the dominant parameter in determining the perceived hardness, which agreed with the previous reports on the perceptual effect of damped natural vibration. In contrast, the perceptual effect of the viscosity depended on the individual. For approximately half the participants, the viscosity did not directly influence the subjective hardness of an object. Some participants felt that a damped natural vibration with greater viscosity was harder. However, the other participants felt that it was harder with a smaller viscosity, which suggested that the decay rate of the mechanical system was used as a criterion to judge the hardness of an object. Therefore, the results indicated that the vibratory frequency is a general parameter that can be used to specify the subjective hardness, whereas the perceptual influence of the viscosity or decay rate depends on the individual.

Index Terms—Haptics, Hardness perception, Vibration

I. INTRODUCTION

When a human taps an object's surface, they perceive its hardness based on the vibration, which consists mainly of a damped natural vibration. Okamura et al. [1], [2] demonstrated that the damped natural vibration elicited the hardness and material perception of a virtual object using a force display. The material perception based on impulsive vibrations also functions in crinkling fragile objects by foot [3], [4]. Kuchenbecker et al. [5] showed that a more realistic damped vibration that included multiple frequency components was more effective at presenting the hardness. The effective use of a damped natural vibration realizes the rendering of an impact that is perceived as a contact with an object harder than what can be perceived with the force feedback methods based on the penalty against penetration between two rigid bodies or the deformation of an object when being pushed [5], [6]. Such

vibration also provides the holder of a long object such as a beam or stick information about the point of contact [7], [8].

The damped natural vibration is characterized not only by the stiffness of the object but also by its mass and viscosity. Therefore, these multiple types of mechanical parameters influence the hardness perception. However, previous studies have not investigated their influence on hardness perception. According to previous studies [1], [2], [5], [6], [9], it is evident that higher frequencies of vibration lead to greater perceived hardness values. Meanwhile, the effect of the decay rate, or time constant, has been unclear. As mentioned later, this depends mostly on the mass and viscosity of an object. Thus far, the effects of the mass and viscosity of an object on the hardness experienced as the result of a damped natural frequency have not been clarified.

This study was performed to determine the contributions of the decay rate and vibration frequency of a damped natural vibration and mechanical parameters, including mass, viscosity, and stiffness, to the hardness perception. We modeled the damped natural vibration as the response of a spring-mass-damper system with one degree of freedom (DOF). We then presented damped natural vibrations with different mechanical parameters using a commercial force display and researched the relationships between the vibration and hardness perception in two experiments. In the first experiment, participants reported the degree of perceived hardness based on the magnitude estimation method. We then discussed the relationships between the subjective hardness and mechanical parameters. In the magnitude estimation method, the definition of perceived hardness was not necessarily shared with the participants. Hence, we also researched the subjective hardness without using such a descriptor in the second experiment. We specified sets of parameters that produced subjectively equal perceived hardness values using the method of adjustment. Based on the results of these two experiments, we analyzed the contributions of the mechanical parameters to the hardness perception.

II. DAMPED NATURAL VIBRATION

We defined an object to be tapped as a 1-DOF spring-mass-damper system. The damped natural vibration was modeled as its step response including the influence of gravity. As shown

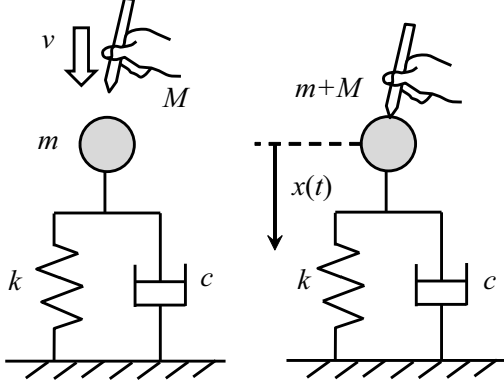


Fig. 1. Tapping of object modeled as 1-DOF spring-mass-damper system.

in Fig. 1, in this system, a hand and stylus with a total mass of M collides with an object that is characterized by mass m , viscosity c , and stiffness k at velocity $v(t)$. The mass and stylus then collectively vibrate as one object with a mass of $m' (= m+M)$. By superimposing the impulse response caused by the momentum of the contact object $Mv(t)$ and the step response caused by the gravity load Mg , the displacement of the vibration at time t , with $t = 0$ being the moment of impact, is represented by

$$x(t) = \frac{Mg}{k} + A \exp\left(\frac{-t}{\tau}\right) \sin(\omega t - \phi) \quad (1)$$

$$A = 2M \sqrt{\frac{v^2 k - vgc + g^2 m'}{k(4m'k - c^2)}} \quad (2)$$

$$\tau = \frac{2m'}{c} \quad (3)$$

$$\omega = \frac{\sqrt{4m'k - c^2}}{2m'} \quad (4)$$

$$\phi = \tan^{-1} \frac{g\sqrt{4m'k - c^2}}{2vk - gc} \quad (5)$$

where A , τ , ω , and ϕ are the amplitude, time constant, vibration frequency, and phase offset, respectively. Because ω and τ rely on m or c as in (3) and (4), the perceptual quality of the damped natural vibration is characterized not only by the stiffness of the object but also by its mass and viscosity.

III. EXPERIMENTAL SETUP AND CONDITIONS

A participant manipulated a commercial force display, Phantom Omni (SensAble), as shown in Fig. 2 and tapped a virtual surface that was located parallel to the table. When the surface was tapped, the force display, with the stylus controlled by a PI controller, presented the displacement of the damped natural vibration represented by (1). The mass, viscosity, and stiffness of the virtual object were variable. The contact speed, which affects the amplitude of vibration was limited to control the experimental condition, which allowed us to focus on the contribution of mechanical parameters. If a participant tapped the virtual object at a speed that was outside the range of 0.15–0.35 m/s, no vibration was presented to them, in order

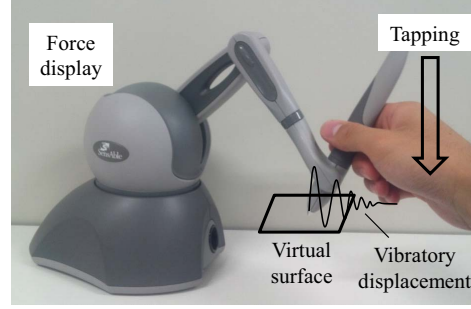


Fig. 2. Experimental apparatus. The subject gripped the stylus of a force display and tapped a virtual surface with variable mechanical parameters.

TABLE I
PARAMETERS OF STIMULI USED IN EXPERIMENT 1

	m [kg]	c [Ns/m]	k [N/m]
Reference Stimulus	0.07	0.60	5000
	0.01	0.60	5000
Test Stimulus	0.04	0.60	5000
	0.10	0.60	5000
	0.13	0.60	5000
	0.07	0.30	5000
	0.07	0.45	5000
	0.07	0.75	5000
	0.07	0.90	5000
	0.07	0.60	2000
	0.07	0.60	3500
	0.07	0.60	6500
	0.07	0.60	8000

to prevent a large variation in the contact speed among the participants and trials. The mass of the contact object (M) was approximated to be 0.1 kg by measuring the weight of the stylus held by a participant that relaxed with his elbow on the table. In the experiments of this study, each participant held a stylus in their writing hand and placed their elbow on a desk with a cushion between them. A reference stimulus was presented on the left half of the force display's work space, and a test stimulus was presented on the right half. Participants were allowed to freely experience the reference and test stimuli during a trial. They wore headphones playing pink noise to block out any auditory cues.

An important point of the experimental design was that the force display delivered the vibratory displacement defined by (1), which did not include the reaction force of the virtual plane. No force feedbacks were presented based on the deformation of the plane or the penalty against the penetration of the stylus through the plane. In this way, we could investigate the effects of the damped natural vibration while decoupling the quasi-static force feedback related to the mechanical properties of the virtual object.

IV. EXPERIMENT

A. Experiment 1: Specification of perceived hardness by magnitude estimation method

1) *Participants and tasks:* The experiment involved five naive participants that signed a written informed consent. Each

participant compared the hardness perceived by the reference and test stimuli and reported the degree of perceived hardness of each test stimulus based on the method of magnitude estimation with the reference stimulus being a modulus. The thirteen test stimuli were presented in random order in a single set. A total of three sets were performed for each participant with a 5-min break between sets.

2) *Parameters of stimuli*: Table I lists the mechanical parameters used for the reference and test stimuli. The parameters for the reference stimulus were set to $m = 0.07$ kg, $c = 0.6$ Ns/m and $k = 5000$ N/m. A value of 10 was designated as the magnitude of its hardness as a modulus. The parameters of the test stimuli were varied using ranges of 0.01–0.13 kg, 0.3–0.9 Ns/m, and 2000–8000 N/m for m , c , and k , respectively. The test stimuli included one whose parameters were the same as those of the reference stimulus. These parameters of stimuli were determined as to be within the stably controllable range of the force display. As a result of these parameter sets, the vibratory frequency, time constant, and amplitude varied within 15–35 Hz, 0.3–0.9 s, 0.8–1.6 mm respectively. In terms of the frequency, some of the earlier studies used two–three times higher than our value [1], [2], [3]. Especially, Ikeda and Hasegawa [7] used a 300 Hz vibration using a high performance haptic display.

3) *Data analysis*: Trials that ended in a remarkable discrepancy between the measured and set displacements of the stylus were excluded from the statistics. The allowable maximum error was set to 0.5 mm at RMS. Such vibratory displacement errors might have occurred as a result of the irregular dynamism of a human hand. The reported values of hardness were geometrically averaged for individuals, and the mechanical parameters were normalized to z -scores. Using a least-squares method, these values were then fitted to a curved surface $H(m, c, k)$, which presented the subjective hardness as a function of the mechanical parameters. The function $H(m, c, k)$ was denoted by

$$H(m, c, k) = p_1(m - p_2)^2 + p_3(c - p_4)^2 + p_5(k - p_6)^2 + p_7 \quad (6)$$

where p_i were regression coefficients. The data acquired through experiment 1 were well fitted with the sufficient coefficients of determination (R^2): 0.83, 0.98, 0.93, 0.98, and 0.89 for participants A–E, respectively.

B. Experiment 2: Investigation of subjectively equal hardness using method of adjustment

1) *Participants and tasks*: This experiment involved five naive participants, two of which also participated in experiment 1. Each participant adjusted the k value of the test stimulus so that the perceived hardness was equal to that perceived using the reference stimulus. The k values were adjusted using a keyboard within a range of 100–10000 N/m, for which the variable pitch was 100 N/m. The eight types of test stimuli were presented in random order in a single set. A total of three sets were performed for each participant, with a 5-min break between sets.

TABLE II
PARAMETERS OF STIMULI USED IN EXPERIMENT 2

	m [kg]	c [Ns/m]	k [N/m]
Reference Stimulus	0.030	0.50	5000
	0.015	0.50	Adj.
	0.020	0.39	Adj.
	0.020	0.61	Adj.
Test Stimulus	0.030	0.35	Adj.
	0.030	0.65	Adj.
	0.041	0.39	Adj.
	0.041	0.61	Adj.
	0.045	0.50	Adj.

Adj.: Adjusted by participants.

TABLE III
GRADIENT OF H IN m , c , AND k DIRECTIONS

Participant	$\frac{\partial H}{\partial m}$	$\frac{\partial H}{\partial c}$	$\frac{\partial H}{\partial k}$
A	-1.49	0.49	1.35*
B	-1.07*	0.71*	1.02*
C	-0.94*	-0.82*	0.80*
D	-2.07*	1.66*	1.92*
E	-4.12*	-0.43	2.30*

* means significant slope at $p < 0.05$.

2) *Parameters of stimuli*: The parameters for the reference stimulus were set to $m = 0.03$ kg, $c = 0.5$ Ns/m, and $k = 5000$ N/m. The parameters of the eight types of test stimuli were set in the vicinity of those for the reference stimulus, as listed in Table II. For the test stimuli, the m and c values were varied in the ranges of 0.015–0.045 kg and 0.35–0.65 Ns/m, respectively. These parameters of stimuli were determined as to be within the stably controllable range of the force display.

3) *Data analysis*: As was done for experiment 1, trials that ended in a remarkable position error between the measured and set displacements of the stylus were excluded from the statistics. The sets of parameters acquired from each participant that were considered to provide subjectively equal hardness values were normalized to z -scores. These values were then fitted to a curved surface on the m - c - k space using a least-squares method. The equation of the surface was defined by a quadratic polynomial function of m , c , and k . This function described the acquired data with coefficients of determination (R^2) of 0.85, 0.62, 0.94, 0.59, and 0.81 for participants D–H, respectively.

V. RESULTS

A. Gradients of H along each mechanical parameter from experiment 1

In order to analyze the effects of the mechanical parameters on the subjective hardness, we computed the gradients of H in the m , c , and k directions in the vicinity of the reference stimulus for each participant. Table III lists the gradients of H in the directions of m , c , and k for each participant in experiment 1. Statistic significance in the table is the significance of regression coefficients. In addition, Fig. 3 shows the curved surface of H established from one participant.

For all the participants, the gradients of H along k were positive, which indicated that the subjective hardness increased as the stiffness of the object increased. In this study, the force

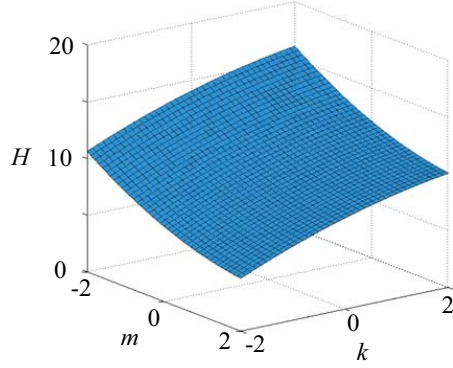


Fig. 3. Subjective hardness H as function of normalized m , k , and c . A sample of participant A in experiment 1 when the c value was fixed at 0.60 Ns/m. H increased as k increased and m decreased.

TABLE IV
GRADIENT OF k IN m AND c DIRECTIONS

Experiment 1		Experiment 2	
Participant	$\frac{\partial k}{\partial m}$	Participant	$\frac{\partial k}{\partial c}$
A	0.73*	F	0.94*
B	0.97*	G	0.64
C	1.08*	H	0.66*
D	1.36*	D	0.43
E	1.04	E	0.64

* means significant slope at $p < 0.05$.

was not fed back based on the deformation of the virtual plane. Only the damped natural vibration was delivered to the participant. Even under this condition, k was found to be the dominant parameter when determining the subjective hardness.

Furthermore, the gradients of H along m were negative for all the participants. As discussed later, these negative gradients indicated the dependence of the hardness perception on the frequency of the damped natural vibration.

In contrast, the gradients of H along c varied among the participants. For two of the five participants, statistically positive gradients were observed, whereas one participant had a negative gradient. For the other two participants, not statistically valid gradients along c were exhibited. Such individual characteristics of c for the hardness experienced as a result of a damped natural vibration are different from those experienced by pushing a deformable object. In the case of the hardness perceived by pushing a deformable object, an increase in c simply leads to the increase in the perceived hardness. However, the effect of c is completely different in the case of vibration-based hardness perception. To the authors' best knowledge, this aspect of human hardness perception has not been previously reported.

B. Gradients of k along m and c

Fig. 4 shows the values reported by one participant (participant F) in experiment 2, and the surface of subjectively equal hardness that was established by the reported values. For this participant, k and m were highly dependent on each other. As k increased, m also increased. In contrast, c was insignificantly

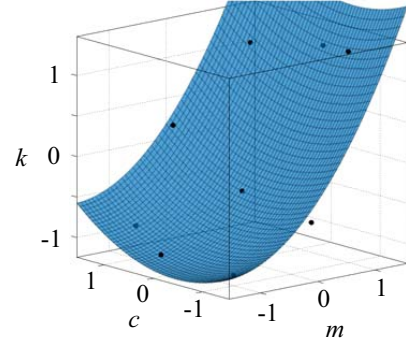


Fig. 4. Surface of subjectively equal hardness in normalized m - c - k space. The results for participant F in experiment 2.

associated with the changes in the k value.

Given that k is an indicator of physical and perceived hardness, we computed the gradients of k along m and c in the vicinity of the reference stimulus for each experiment. For this computation, the k values of samples in experiment 1 were fitted to a polynomial quadratic function of m , c , and H . The gradients were then computed around the m , c , H values of the reference stimulus used in experiment 2. Table IV lists these gradients. Positive gradients of k along m were observed for all the participants. In other words, to maintain the subjective hardness, k and m varied in the same direction, which is in line with the idea that the hardness perceived by damped natural frequencies is influenced by its vibratory frequency, as discussed later. In contrast, the signs of the gradients of k along c varied among the participants. The gradient was significantly positive for two participants, but it was negative for two participants for both types of experiments. The other participants did not exhibit any statistically significant gradients. Such an individuality for the effect of c was consistent in the two experiments.

VI. DISCUSSION

As listed in Table IV, the gradients from participants D and E were consistent between the two types of experiments. Thus, we could reach similar conclusions in both types of experiments.

A. Significant effect of vibratory frequency on hardness perception

A positive gradient of k along m indicated the dependence of the hardness perception on the frequency of the damped natural vibration. As shown in (4), the frequency is substantially proportional to $(k/m)^{\frac{1}{2}}$. The frequency remains constant when increasing m and k simultaneously. Such a hardness perception based on the vibration frequency is consistent with the results of previous research [1], [2], [5], [6], [9], where a higher frequency led to a greater subjective hardness. Furthermore, this effect of the frequency was rigorous for all the participants, because the positive gradients were statistically significant for most of the participants.

B. Individual effect of c or decay rate on hardness perception

The gradients of k along c varied among the participants. Hence, we could not reach a general conclusion about the relationship between k and c or the effect of c on the perceived hardness.

For two and three of the five participants in experiments 1 and 2, respectively, the gradients were insignificant, which indicated that c was not significantly related to k . This indicated that for some participants, the subjective hardness based on the damped natural vibration did not rely on the viscosity of the object, or the hardness originating from k qualitatively differed from that originating from c .

For one participant in each type of experiment, the gradient of k along c was significantly positive. For these participants, the decay rate might have become a cue for hardness perception because the decay rate of 1-DOF spring-mass-damper system is affected by the ratio of c to $k^{\frac{1}{2}}$ as follows:

$$\zeta = \frac{c}{2\sqrt{mk}}. \quad (7)$$

For the other participants, the gradient of k along c was negative, suggesting that an increase in c led to a greater subjective hardness. This was because the negative gradient meant that the increase in c was needed to compensate for the decrease in k to maintain the subjective hardness. This appears to be in line with the hardness perception of a deformable object by pushing, where an increase in c also increases the perceived hardness. For these participants, the damped natural vibration might have been linked with the mechanical impedance of the object, and the mechanical impedance played a greater role in the hardness perception rather than the decay rate.

In summary, although the reasons remain to be studied, a change in the value of c influenced individuals differently. This indicates that we should not expect the decay rate to have similar effects on individuals in terms of the perceived hardness.

VII. CONCLUSIONS

A hardness presentation based on a damped natural vibration effectively delivers the sense of hardness. Furthermore, this method is applicable not only for force displays but also for vibrotactile displays. Hence, it is expected to become a general technique and be widely used for mobile devices. In general, the vibratory frequency is known to regulate the hardness experienced as a result of a damped natural vibration. However, the effects of other types of mechanical parameters have yet to be determined. We investigated their contributions using a commercial haptic display. In the experiments, the damped natural vibration caused by the contact between two bodies was presented, and no force feedback related to the penetration of the tip of a stylus to the virtual object or its deformation was conveyed. In this way, we could purely investigate the perceptual influence of the vibration.

In the two types of experiments using the methods of magnitude estimation and adjustment, the perceptual influences

of the stiffness and mass of the virtual object were prominent and shared among the participants. An increase in the stiffness resulted in an increase in the perceived hardness. An increase in the mass led to a decrease in the perceived hardness. These results support the conclusion that the vibratory frequency strongly influences the hardness perceptions, which is consistent with the results of previous studies.

In contrast, individual differences were observed in the effects of the viscosity of the virtual object. For approximately half the participants, changes in the viscosity were not accompanied by changes in the perceived hardness, suggesting an insignificant effect of the viscosity on the perceived hardness. For some participants, an increase in the viscosity decreased the subjective hardness. Such an effect makes sense, considering that the decay rate plays a role as a perceptual cue. For the rest of the participants, an increase in the viscosity increased the subjective hardness, which is in line with the perceptual phenomenon held for deformation- or penetration-based object rendering. Therefore, the effects of the viscosity could substantially vary across individuals.

Because of the limited abilities of the commercial force display used in this study, these results were based on a narrow range of vibratory stimuli. For more general aspects, further studies with wider range of vibratory stimuli and more participants are necessary. Nonetheless, to summarize the results of this study, the hardness presented by a damped natural vibration could be adjusted by changing the frequency of the vibration because its perceptual effects were prominent for the majority of those who experienced the stimuli. However, the effect of the decay rate or viscosity on the hardness perception may vary among people, whereas its effect on material perception is not denied.

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REFERENCES

- [1] A. M. Okamura, M. W. Hage, M. R. Cutkosky, and J. T. Dennerlein, "Improving reality-based models for vibration feedback," Proceedings of ASME Dynamic Systems & Control Conference, vol. 69, no. 2, pp. 1117–1124, 2000.
- [2] A. M. Okamura, M. R. Cutkosky, and J. T. Dennerlein, "Reality-based models for vibration feedback in virtual environments," IEEE/ASME Transactions on Mechatronics, vol. 6, no. 3, pp. 245–252, 2001.
- [3] S. Okamoto, S. Ishikawa, H. Nagano, and Y. Yamada, "Spectrum-based vibrotactile footstep-display for crinkle of fragile structures," Proceedings of IEEE International Conference on Robotics and Biomimetics, pp. 2459–2464, 2011.
- [4] S. Okamoto, S. Ishikawa, H. Nagano, and Y. Yamada, "Spectrum-based synthesis of vibrotactile stimuli: Active footstep display for crinkle of fragile structures," Virtual Reality, vol. 17, no. 3, pp. 181–191, 2013.
- [5] K. J. Kuchenbecker, J. Fiene, and G. Niemeyer, "Improving contact realism through event-based haptic feedback," IEEE Transactions on Visualization and Computer Graphics, vol. 12, no. 2, pp. 219–230, 2006.
- [6] S. Hasegawa, Y. Takehana, A. Balandra, H. Mitake, K. Akahane, and M. Sato, "Vibration and subsequent collision simulation of finger and object for haptic rendering," Haptics: Neuroscience, Devices, Modeling, and Applications, Part II, M. Auvray and C. Duriez Eds., Springer Berlin Heidelberg, pp. 352–359, 2014.

- [7] R. Okazaki and H. Kajimoto, "Altering distance perception from hitting with a stick by superimposing vibration to holding hand," *Haptics: Neuroscience, Devices, Modeling, and Applications*, M. Auvray and C. Duriez Eds., Springer Berlin Heidelberg, pp. 112–119, 2014.
- [8] J. Sreng, A. Lecuyer, and C. Andriot, "Using vibration patterns to provide impact position information in haptic manipulation of virtual objects," *Haptics: perception, devices and scenarios*, Springer Berlin Heidelberg, M. Ferre Ed., pp. 589–598, 2008.
- [9] Y. Ikeda and S. Hasegawa, "Characteristics of Perception of Stiffness by varied tapping velocity and penetration in using event-based haptic," *Proceedings of 15th Joint Virtual Reality Eurographics Conference on Virtual Environments*, pp. 113–116, 2009.