Interactive Forces Caused by Scanning Wavy Surfaces
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Abstract
Well-designed force fields can cause the perception of surface shapes such as holes or bumps on a flat surface without involving actual geometric displacements during exploratory hand motions. To establish a better force field model, we observed the interactive forces caused by scanning a sinusoidal wavy surface under point and surface contact conditions. Under the point contact condition, the acquired forces fit well into the model of earlier studies with the exception of kinetic frictions. When the surface was scanned with a bare finger to obtain a surface contact area, the observed force profiles significantly deformed compared with those of the earlier static point-contact model. Such deformation of the force profile can be explained by combining Hertz contact theory and the static point-contact model. The results of this study will lead to the development of an interactive force model for surface exploration, that is expected to improve force rendering algorithms.

1 Introduction
A well-designed force field on a flat plane can produce the perception of displacements perpendicular to the plane [1, 2]. For example, Fig. 1 shows that delivering assistive or resistive forces to a person’s finger causes the sense of shape along Y-axis as the finger explores the X-Z plane. Such force fields can induce virtual surface profiles that are perceptually equivalent to real ones [2, 3]. The expression of surface profiles using the force field on a plane is compatible with prevalent pointing interfaces represented by a mouse and touch panel. If these 2D or planar interfaces are equipped with force feedback functions, they can allow users to feel the three-dimensional characteristics of surfaces (e.g., [4, 5]), which can potentially become a major industrial or commercial application of haptic interfaces.

The fact that a force field on a plane evokes the perception of surface profiles is an important finding in terms of both practical applications of haptic interfaces and understanding human perceptual mechanisms. Improving this technique is commonly beneficial for haptic researchers. There are still some gaps between earlier force field models and the actual phenomenon; there are the result of the complex interaction between a soft finger pad and geometric shapes. In this study, we focused on one of the most successful models used by Robles-De-La-Torre and Hayward [2]. As described in Sec. 2, this model assumes point and frictionless contact. These assumptions are close to the conditions when a human slowly scans a surface profile with a frictionless pointing probe such as a ball bearing. However, they may somewhat differ from those when exploration is carried out with a human finger pad.

The objective of this study was to observe interactive forces caused by exploration of a wavy surface profile under two contact conditions (point and surface contact) and two kinetic conditions (pseudo static and dynamic). We compared the observed force vectors with the earlier virtual shape model and specified the effects of the exploring conditions.

2 Static Point-Contact Model
As an example of the presentation of virtual shapes, Robles-De-La-Torre and Hayward successfully demonstrated virtual bumps or holes using force fields on a plane [2]. In this method, they geometrically expanded the force vector received by a person from the surface when s/he scans it with a probe. As shown in Fig. 2, the horizontal force depends on the vertical force or gradient of the surface. In this figure, $f_x$ and $f_y$ are the force that a person applies to the surface and its reaction force, respectively. When a probe or contactor remains still on the surface at $(x, y)$, the components of $f_p$ satisfy

$$f_{px} = f_{py} \tan(\alpha(x))$$

where $\alpha(x)$ is the angle of the surface at $x$. The relationship between the surface profile function $y(x)$ and this angle is

$$\tan(\alpha(x)) = \frac{dy}{dx}.$$ (2)

Hence, the ratio of force components is equal to the gradient of the surface profile, which is described by

$$\frac{f_{xx}}{f_{yy}} = \frac{dy}{dx}.$$ (3)

Using the above force equilibrium, a force field can evoke the perception of surface displacements on a flat surface. The force presented by a haptic display is denoted by $f_v$ and should satisfy $f_x + f_y + f_p = 0$. The X-axis component of this equation is $f_{vx} + f_{vy} = 0$ because $f_{py} = 0$ on the flat surface. The Y-axis component is $f_{xy} + f_{yy} = 0$ because $f_{xy} = 0$ in the case of a 2-d.o.f. force display as shown in Fig. 1. Instead of $f_{px}$, which is the reactive force from the angled surface, the force field $f_{vx}$ is introduced. The horizontal force presented by the force displays is then determined by

$$f_{vx} = f_{py} \tan(\alpha(x)).$$ (4)

This force rendering algorithm successfully leads to the perception of wavy surfaces such as bumps or holes where the profiles are defined by $\alpha(x)$. Similar force rendering algorithms are also effective.
for presenting surface curvature in 3D environments involving actual displacements perpendicular to the surface [6].

There are some concerns when we compare the above force rendering algorithm and the phenomenon caused by interaction between a finger pad and wavy surface. First, the contact between a finger pad and surface produces a contact area. Although the reactive force centers at a point of contact in the above algorithm, the pressure diffuses within the area contribute to the curvature percepts [7, 8]. Second, the above model does not include friction that naturally exists between two objects. Such friction should be fairly considered because the significant effect of friction on the perception of curvature is known [9]. These differences between the model and finger pad’s contact potentially lead to a substantial difference in the relationships of related force vectors.

3 EXPERIMENT

We observed the forces produced when a sinusoidal surface was explored. In order to determine the effects of the contact conditions, we compared the explorations by a finger pad and ball bearing with moderately frictionless point contact. The exploratory speed was controlled at several different levels to investigate its influence on the interactive forces.

3.1 Equipment

Fig. 3 shows the experimental setup: the position of the finger or ball bearing and the interactive forces between it and the sinusoidal surface were recorded at 1 kHz. A triaxial force sensor (USL06-H5, Tec Gihan Co. Ltd., nominal resolution: $19.6 \times 10^{-3}$ N) located beneath the surface measured the X- and Y-axis forces applied to the surface. The position of the finger or bearing was measured using two encoders (RE30E-500, Nidec Copal Electronics Corp., 500 ppr): each was attached to a string that was fixed through a pulley with a diameter of 13 mm. The encoders were placed at the height of $h = 250$ mm with the interval $w = 500$ mm. The resultant measurement resolution were 14.4 and 17.2 μm along the X- and Y-axes, respectively. The end of the string was fastened to the tip of the ball bearing or to a ring worn on the index finger. The diameter of the ball bearing was 0.5 mm, and it was well-lubricated during the experiments.

3.2 Sinusoidal surface

The sinusoidal wavy surface used in the experiment was made of an ABS plastic board (170 mm × 50 mm). The interval between two waves was set to 40 mm, and the height difference between the peak and valley was 5 mm. The surface displacement along $x$ was

$$y(x) = 2.5 \sin \left(\frac{2\pi x}{40}\right)$$

in millimeters. The surface was polished with a fine sand cloth. The participants coated their finger pads with talcum powder before scanning the surface. This powder effectively prevented the finger pads from sticking to the surface because of perspiration.

3.3 Procedures

One volunteer continuously explored the surface for 120 s while attempting to maintain contact between the finger pad or ball bearing and the surface. Even if the contact was not maintained, irregularities were easily detected from the output of the force sensor. When the volunteer used the ball bearing, he tried to maintain the posture of its grip handle perpendicular to the desktop. When exploring with a bare finger, he again maintained the posture of his hand as much as possible. During the exploration, he was instructed to relax so that extraordinarily large forces were not applied to the surface. To observe the effect of scanning speeds i.e. quasi static or dynamics, three reference hand speed levels were used for exploration with the ball bearing: 10, 30, and 60 mm/s. In contrast, only one reference speed was used for finger exploration: 30 mm/s. As described later, during analysis we excluded force samples observed at speeds significantly far from these reference speeds.

4 RESULTS

4.1 Data analysis

The positions of fingertip or ball bearing were computed from the data acquired by the encoders. The position $(x, y)$ should satisfy

$$\begin{cases} 
\left(\frac{w}{2} + x\right)^2 + (h - y)^2 = l_1^2 \\
\left(\frac{w}{2} - x\right)^2 + (h - y)^2 = l_2^2
\end{cases}$$

where $l_1$ and $l_2$ are the distance between each encoder and ball bearing or fingertip. The velocities of the fingertip and ball bearing were computed from the position data. We classified the ball bearing velocities on X-Y plane into three levels: 0–20, 20–40, and 40–100 mm/s. Those above 100 mm/s were not used for the analysis. For the fingertip scan, we only used samples with velocities of 30–50 mm/s. Because we were interested in the equilibrium of force ratios as determined by (3), we computed the force ratios $f_s(x)/f_x(x)$ along the X-axis after excluding outliers with weak $f_y$—owing to irregular
contact. Naturally, these ratios fluctuated, as shown in Fig. 4. We averaged them to determine the final values.

4.2 Exploration by ball bearing (pseudo point contact and low friction)

Fig. 5 shows the force ratios observed when the ball bearing moved on the surface from left to right. These figures show the data for 60 mm just above the force sensor. The top figure is the profile of the scanned surface, $y(x)$. The middle and bottom ones are the force ratios, $f_\text{x}/f_\text{y}$. All figures share the horizontal position axis.

We first examine the middle figure with two curves: curve determined by the model of (3), and the experimentally observed curve as the ball bearing explored the surface at the speed of less than 20 mm/s. The profiles of the two curves were fairly similar, which means that (3) functions in the case of a ball bearing that was close to the low-frictional point contact condition.

The exception between the model and observed curves was the maximum peak values of the force ratios. The peak force ratio of the observed data was larger than that of the model curve, as shown by the double-headed arrow in the figure. This gap was due to the friction inside of the ball bearing. Although we considered the exploration by ball bearing to be frictionless, tangential force increased by friction.

The bottom of Fig. 5 shows the three observed force ratios at different velocity levels. Considering the sampling deviation, we concluded that there were no substantial differences among these three curves despite the different scanning speeds. Within the range of our experimental conditions, the ratios of the horizontal and normal forces did not depend on the probe speeds.

4.3 Exploration by index finger (surface contact with friction)

Fig. 6 shows the force ratios observed when the surface was explored by the index finger for which frictional surface contact was assumed. The gray and black curves represent those observed when the finger moved rightward and leftward, respectively. The dotted curve was determined by the static point-contact model of (3). These curves significantly differed from each other unlike those observed using the ball bearing.

First, the base levels of the curves shifted. Compared with the static point-contact model (dotted curve), the gray curve significantly shifted upward. Similarly, the black curve shifted downward. These base-line shifts were due to the friction forces, which increased the interactive forces along the moving direction. When the kinetic friction was effective, the X-axis component of the interactive forces increased as described above.

Second, the profiles of the force ratios substantially deformed, and they were no longer similar to cosine curves. For the static point-contact model (dotted curve), the peak of the force ratio was at the maximum gradient of the surface. However, the peak of the observed curve (gray curve) shifted leftward compared with that of the dotted curve. Similarly, the bottom of the gray curve shifted rightward compared with that of the dotted curve. Such deformation of the force ratio curves was due to the surface contact of the finger pad. For the point contact model, the reaction force was solely applied to the contact point; however, with surface contact, the pressure distribution in the contact area varied depending on the gradient of the surface. We discuss the deformed profile of the force ratio curves in the next section.

5 Skewed profile of force ratio caused by asymmetric pressure distribution within contact surface

5.1 Asymmetry of pressure distribution in contact area

According to the Hertz contact theory [10], a circular contact surface with a diameter of $2a$ is obtained from the contact of two spherical objects. As shown in Fig. 7, the pressure is distributed on the area in a parabolic manner with the maximum pressure $p_{\text{max}}$ at the center of the area. $R$ is the radius of the object. The pressure at $r$
Their ratio becomes
\[ p \] from the circular center is expressed as
dex finger at 30–50 mm/s
Figure 6: Force component ratios when surface was scanned by in-
\[ \frac{f_x}{f_y} \] metric pressure distribution within the contact area. As shown in
\[ R \] the other side of the surface that is closer to the valley of the wavy
\[ p \] closer to the top of the bump, the pressures are larger than those on
\[ p \] With a positive angle of the surface,
\[ p \] (8)
\[ \sin(\alpha) \]1
\[ \tan(\alpha(x-r)) \]1
\[ \tan(\alpha(x+r)) \]
\[ \sin(\alpha) \]
\[ \sqrt{1 - \left( \frac{-r - R \sin(\alpha)}{a} \right)^2} \times \]
\[ \sqrt{1 - \left( \frac{r - R \sin(\alpha)}{a} \right)^2} \times \tan(\alpha(x-r)) \plus \]
\[ \sqrt{1 - \left( \frac{r - R \sin(\alpha)}{a} \right)^2} \times \tan(\alpha(x+r)) \}
\[ \frac{f_x}{f_y} = \frac{1}{\sqrt{1 - \left( \frac{-r - R \sin(\alpha)}{a} \right)^2} + \sqrt{1 - \left( \frac{r - R \sin(\alpha)}{a} \right)^2} \times \}
\[ \{ \sqrt{1 - \left( \frac{-r - R \sin(\alpha)}{a} \right)^2} \times \tan(\alpha(x-r)) \plus \]
\[ \sqrt{1 - \left( \frac{r - R \sin(\alpha)}{a} \right)^2} \times \tan(\alpha(x+r)) \}

Using (9), the ratio of asymmetric pressures at two points, the force
\[ \frac{f_x}{f_y} \] will lead to better force rendering techniques in the future.
\[ f \] Taking account of the dimensions of a human finger, we determined
\[ f \] the pertinent parameters such that the profile of this force ratio re-
\[ f \] sembles the observed ones. The parameters were set to \[ R = 7 \],
\[ f \] \[ r = 3.4 \], and \[ a = 6 \] mm. Fig. 9 shows the one from this equa-
\[ f \] tion and those from the exploration using the index finger that are
\[ f \] the same as those in Fig. 6. The above equation fairly expresses
\[ f \] the phase shifts of their peaks and the profile that resembles the
\[ f \] observed force ratio curves.

Fig. 10 shows the effect of the parameter. Larger \( r \) values deform
\[ f \] the profile of force ratios more because \( r \) determines the degree
\[ f \] of asymmetry of pressures. From this simulation, we suggest that the
\[ f \] skew of the force profile is mainly due to the asymmetric pressure
\[ f \] distribution within the finger pad. Still, the base levels of these
\[ f \] simulated and observed curves are largely different. As discussed
\[ f \] before, such base line shifts are potentially explained by kinetic
\[ f \] friction. Hence, the differences in the profiles of force ratios be-
\[ f \] tween the static point-contact model and the exploration of finger
\[ f \] pad mainly originate from the surface contact and friction.

6 CONCLUSION

A well-designed planar force field evokes the perception of shapes
\[ f \] with displacement perpendicular to the plane. This is a beneficial
\[ f \] perceptual mechanism that allows 3D shapes to be presented by
\[ f \] haptic displays with limited degrees of freedom. Establishing bet-
\[ f \] ter algorithms to produce force fields based on the observation of
\[ f \] actual haptic exploration is an important challenge. In this study,
\[ f \] we observed interactive forces when a sinusoidally displaced sur-
\[ f \] face was scanned at different contact conditions and speeds. When
\[ f \] the surface was explored with a ball bearing, the profiles of the ra-
\[ f \] tios of horizontal and vertical forces were similar to those of the
\[ f \] static point-contact model with the exception of larger force ratios
\[ f \] originating from friction at the rising slope. We did not observe any
\[ f \] differences in the force ratios among the different speed levels of
\[ f \] exploration. When the surface was explored with a bare finger, the
\[ f \] profile of the force ratios substantially deformed. Such deformation
\[ f \] can be explained by the surface contact between the finger pad and
\[ f \] surface. An asymmetrical pressure distribution on the contact area
\[ f \] was produced by the inclined surface, which skewed the force ratio
\[ f \] profile. Understanding the force interaction of finger exploration
\[ f \] will lead to better force rendering techniques in the future.
REFERENCES


